

On using ARIMA model confidence intervals applied to population projections based on the components of change: a case study for the world population

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Abstract

This paper shows how measures of uncertainty from a standard time series model (ARIMA) can be applied to an existing population projection based on components of change using the world as a case study. The measures of forecast uncertainty are relatively easy to calculate and meet several important criteria used by demographers who routinely generate population forecasts. This paper applies the uncertainty measures to a world population forecast based on the Cohort-Component Method. This approach links the probabilistic world forecast uncertainty to the fundamental demographic equation, the cornerstone of demographic theory, which is an important consideration in developing accurate forecasts. The results are compared to the Bayesian probabilistic world forecast developed by the United Nations and found to be similar but show more uncertainty. The results are followed by a discussion suggesting that this new method is well-suited for developing probabilistic world, national, and sub-national population forecasts.

Key words: ARIMA, Bayes, Espenshade-Tayman method, forecast uncertainty, super population.

1. Introduction

Alkema *et al.* (2015) describe a Bayesian approach that links probabilistic uncertainty to a world population forecast based on the Cohort-Component Method (CCM). It proceeds by assembling a large sample of future trajectories for an outcome such as the total population size. The point projection in a given year is the median outcome of the sample trajectories. Other percentiles are used to construct prediction intervals (Alkema *et al.*, 2015). More details on this seminal approach are found in Raftery, Alkema, and Gerland (2014), and a general overview of probabilistic population forecasting can be found in Raftery and Ševčíková (2023).

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Because the Bayesian approach described by Alkema *et al.* (2015) is based on the CCM, its measures of uncertainty are linked to the “fundamental equation”, whereby a population at a given point in time, P_{t+k} , is equal to the population at an earlier point in time, P_t , to which is added the births and in-migrants that occur between time t and time $t+k$ and to which is subtracted the deaths and out-migrants that occur during this same time period (Baker *et al.*, 2017, pp. 251–252). The fundamental equation is the cornerstone of demographic theory and is the foundation upon which the CCM rests (Baker *et al.*, 2017, pp. 22–23; Burch, 2018; Verma, 2023). A probabilistic approach to population forecasting based on this theoretical foundation yields benefits not found in methods lacking this foundation (e.g., Burch, 2018; Land, 1986). This observation is also consistent with one made by Swanson *et al.* (2023), who argue that a given population forecasting method’s strengths and weaknesses largely stem from four sources: (1) its correspondence to the dynamics by which a population moves forward in time; (2) the information available relevant to these dynamics; (3) the time and resources available to assemble relevant information and generate a forecast; and (4) the information needed from the forecast. The Bayes CCM approach comes with strengths. However, it also comes with weaknesses. Goodwin (2015) finds Bayesian inference difficult, effortful, opaque, and even counter-intuitive. Along with the weaknesses described by Goodwin (2015) are implied ones, including being not easy to apply or explain and having a low face validity and high production costs in that a Bayes CCM approach is very data- and analytically intensive.

2. Objective

In view of the facts described in the preceding section, we offer an approach for constructing uncertainty measures that is relatively simple and linked directly to the CCM approach. Importantly, unlike Bayesian inference, we believe it is likely to meet important evaluation criteria used by demographers who routinely develop population forecasts (Smith, Tayman, and Swanson, 2013, pp. 301–322); low production costs (particularly staff time); easy to apply and easy to explain; a high level of face validity; and intuitive.

3. Methods and Data

In describing this new approach, we use a world population forecast with a horizon of 2060. It is found at the *International Data Base* (IDB) site of the U.S. Census Bureau (https://www.census.gov/data-tools/demo/idb/#/table?COUNTRY_YEAR=2024&COUNTRY_YEAR_ANIM=2024&menu=tableViz). The data and methods are documented in U.S. Census Bureau (2020).

The approach we suggest employs the ARIMA (Auto-Regressive Integrated Moving Average) time series method in conjunction with work by Espenshade and Tayman (1982), whereby we can translate the uncertainty information found in the ARIMA method's forecast to the population forecast provided by the CCM approach. We describe neither the ARIMA (Box and Jenkins, 1976) nor the CCM approach (Smith, Tayman, and Swanson, 2013, Chapter 7) in detail because they are widely known and used. However, as described by Smith, Tayman, and Swanson (2001, pp. 172–176), an ARIMA model attempts to uncover the stochastic processes that generate a historical data series. The mechanism of this stochastic process is described—based on the patterns observed in the data series—and that mechanism forms the basis for developing forecasts. Up to three processes can represent the stochastic mechanism: autoregression, differencing, and moving average. The most general ARIMA model is usually written as ARIMA (p, d, q), where p is the order of the autoregression, d is the degree of differencing, and q is the order of the moving average.

In regard to this case study, the patterns of the autocorrelation (ACF) and partial autocorrelation functions (PACF) were used to find the correct values for p and q (Brockwell and Davis, 2016: Chapter 3). The ARIMA model shown here had random residuals and the smallest possible values for p , d , or q , as determined by the Ljung-Box test (Ljung and Box, 1979). We chose an “adequate” ARIMA model using these criteria. We note that there may be other versions that also are “adequate” and that further refinement of the selection process can be done (e.g., using the augmented Dickey-Fuller test (Dickey and Fuller, 1979) to identify the amount of differencing required to achieve a stationary time series). Because our aim here is heuristic and not definitive, we did not pursue further refinement of the ARIMA model we present beyond determining it to be adequate.

Before turning to a description of the new method, we first clarify our use of the term “confidence interval” in regard to forecast uncertainty. It is more common to use the term “forecast interval” or “prediction interval” in the context of forecasting because a “confidence interval,” strictly speaking, applies to a sample (Swanson and Tayman, 2014, p. 204). However, underlying the approach we describe herein is the concept of a “super-population,” which, as discussed later, describes a population that is but one sample of the infinity of populations that will result by chance from the same underlying social and economic cause systems (Deming and Stephan, 1941). The concept of viewing a forecast as a sample leads us to choose the term “confidence interval” rather than forecast interval or prediction interval.

We use annual world historical data of total population and land area in square meters to compute population density annually from 1950 to 2020 found at the IDB site to implement the ARIMA model found in the NCSS statistical package (NCSS, 2024) and launch from the annual world forecasts found at the same site for 2021–2060.

Exhibit 1 contains the NCSS output and report on the ARIMA model we use. We use “density” because the Espenshade-Tayman (1982) method for translating uncertainty information does so from an estimated “rate,” which in this case is the “rate” of population density. Thus, the 95% confidence intervals generated by the ARIMA world “density” forecasts are translated to the CCM-based world population forecast. Other denominators could be used in developing this “rate” such as the ratio of the population to housing units. However, using the land area as the denominator provides a virtually constant denominator over time, thereby reducing the effort in assembling the “rate” data. It also serves as a stabilizing element regarding the use of ARIMA in that it dampens the effect of short-term population fluctuations more effectively than, say, housing units, which also can fluctuate over time and not always in concert with population fluctuations. As should be obvious, the data assembled to develop the ARIMA density forecast should encompass the base data used to develop the population projection in terms of the total population numbers. The case study we present meets this condition in that the ARIMA model covers the annual period from 1950 to 2020 and the population projection data use the total 2020 population, supplemented by earlier data in the examination of trends.

Exhibit 1 About Here

Here is an example of this process using the 2050 world population projection result found at the IDB site.

Let P = projected world population (at time t_i)

Let D = forecasted world population density obtained from ARIMA at time t_i , and

Let A = land area of the world (131, 821, 645 square kilometers).

The 2050 ARIMA density forecast shows 73.02, 76.81, and 80.60 persons per square kilometer, respectively, for the land area of the world as a whole (95% Lower Limit of forecasted D , forecasted D , and 95% Upper Limit of forecasted D , respectively).

The relative widths of the Lower and Upper Limits are -0.04938 and 0.04938, respectively.

The 2050 world population projection found at IDB is 9.7 billion.

Multiplying 9.75 billion by -0.04938 and adding this product to 9.75 billion yields 9.27 billion, the 95% Lower Limit, and adding the product $9.75 \text{ billion} \times 0.04938$ to 9.7 billion yields 10.23 billion, the 95% Upper Limit of the 2050 world population forecast found at IDB.

Putting it all together, we can state that we are 95% certain that the 2050 world forecast found at IDB is between 9.27 billion and 10.23 billion.

As alluded to earlier, underlying the Espenshade-Tayman method is the idea that a sample is taken from a population of interest. In this case, the ARIMA results represent the sample, and the CCM forecasts represent the population. This interpretation is derived from the idea of a “super-population” (Hartley and Sielken, 1975; Sampath, 2005; Swanson and Tayman (2012, pp. 32–33). This concept can be traced back to Deming and Stephan (1941), who observed that even a complete census, for scientific generalizations, describes a population that is but one of the infinity of populations that will result by chance from the same underlying social and economic cause systems. It is a theoretical concept that we use to simplify the application of statistical uncertainty to a population forecast that is considered a statistical model in this context. This approach is conceptually and mathematically different from the classical frequentist theory of finite population sampling (Hartley and Sielken (1975)), but as pointed out by Ding, Li, and Miratrix (2017), in practical terms, these two approaches result in identical variance estimators. As such, we believe that our approach is on solid statistical ground. Before moving on, we also note that using the Espenshade-Tayman method (1982) here is not new. In addition to being employed by Espenshade and Tayman (1982), it has been used by Swanson (1989) and Roe, Swanson, and Carlson (1992) in demographic applications.

4. Results

Table 1 provides population forecast uncertainty measures for the world population from 2020 to 2060 by decade. The table contains three panels. Panel A provides the ARIMA-generated world population density point forecast and their 95% lower and upper limits. Panel B provides the relative widths of these intervals, which measure the proportionate differences between the upper and lower bounds and the point forecast. Panel C provides the IDB world point forecast along with the 95% CIs the Espenshade-Tayman approach has generated for them.

The relative bounds shown in Panel B, ignoring the sign on the LL95%, are analogous to the half-width that measures interval width (Tayman, Smith, and Lin, 2007), but half-widths are expressed as percentages and not proportions. As the name suggests, a half-width is $\frac{1}{2}$ the width of the confidence interval, and we define this measure as $\text{half-width} = ((\text{MOE}/2) / (\text{projection})) \times 100$. As expected, the confidence intervals around the IDB point forecast increase in width as the forecast horizon length increases. The half-width for the 10-year horizon (2030) is 1.6% and rises steadily, reaching 6.3% for the 40-year horizon (2040).

Table 1 About Here

As shown in Table 1, the 2050 world forecast found at IDB is 9.75 billion with a 95% lower bound of 9.27 billion and a 95% upper bound of 10.23 billion. Under its medium scenario, the United Nations (UN) expects a world population in 2050 of 9.7 billion, with a 95% lower bound of 9.4 billion and a 95% upper bound of 10.0 billion (United Nations, 2022a: 28). The IDB-based point forecast for 2050 is 0.5% higher than the UN's medium scenario point forecast for 2050, where $0.5\% = 100 \times (9.75 - 9.70) / 9.70$. The lower bound for the IDB-based 2050 forecast differs by 1.4% from the lower bound of the UN's medium scenario 2050 forecast, where $1.4\% = 100 \times (9.40 - 9.27) / 9.27$ and by -2.25% from the upper bound, where $-2.25\% = 100 \times (10.0 - 10.23) / 10.23$. In other words, for 2050, the UN 95% intervals are narrower than the intervals produced from the IBF using the ARIMA process. The range in 2050 for the UN interval is six hundred million persons, 37 percent lower than the range from the IDB forecast paired with the ARIMA density model (960 million persons).

Although the specific 95% confidence intervals have not been discussed for years other than 2050 in the UN report on its probabilistic world population forecasts, they are available in an Excel file (UN, 2024). We used them to produce "half-widths" for the 2030, 2040, 2050, and 2060 UN forecasts. They are, respectively, 0.53%, 1.49%, 2.51%, and 3.76%; the half widths that we constructed of the IDB forecasts for 2030, 2040, 2050, and 2060 are, respectively, 1.60%, 3.35%, 4.94%, and 6.28%. Although the difference narrows between 2030 and 2060, the uncertainty intervals we constructed for the IDB forecasts are wider than those found for the UN forecasts. However, like the UN uncertainty intervals, our constructed ones increase over time. Between 2030 and 2060, the uncertainty interval we constructed for the IDB forecasts increases almost fourfold while the UN's increase is sevenfold.

5. Discussion

As is the case with the Bayesian approach described by Alkema *et al.* (2015), the new approach we propose can be linked directly to the CCM method (as well as forecasts produced by other methods such as the Cohort Change Ratio (CCR) approach, which is algebraically equivalent to the CCM approach, but requires less input (Baker et al., 2017, pp. 251–252)). Unlike the approach found in Swanson and Beck (1994), neither the CCM nor the CCR approach is inherently conjoined with a method for generating statistical uncertainty. Thus, we believe this linkage represents a step toward generating probabilistic forecasts based on the fundamental population equation. Notably, the ARIMA method is widely available in the software packages generally used by demographers.

Considering the discussion of data assembly and analysis (Alkema *et al.*, 2015; United Nations, 2022; Yu *et al.*, 2023), it is clear that far more time and resources are required for a Bayesian probabilistic forecast than for the probabilistic forecasting method we have described here. In addition, as characterized by Goodwin (2015), Bayesian inference is difficult, effortful, opaque, and even counter-intuitive, none of which applies to the method we have described in this paper. Beyond Goodwin's (2015) observations, Green and Armstrong (2015) discuss simple versus complex methods in terms of forecasting, which applies here in that the approach we describe falls more into the simple methodological category rather than the complex category. They suggest that while no evidence shows complexity improves accuracy, complexity remains popular among (1) researchers because they are rewarded for publishing in highly ranked journals, which favor complexity; (2) methodologists because complex methods can be used to provide information that supports decision makers' plans; and (3) clients who may be reassured by incomprehensibility.

The ARIMA model (ARIMA, (1,1,0)) we selected (see Exhibit 1) is one of several "adequate models" we examined. We found that the other two, ARIMA (1,1,1) and ARIMA (1,2,0), produced uncertainty intervals that varied from the model we selected but were consistent overall in that their 95% confidence intervals also increased over time, which brings up a point of interest. Swanson and Tayman (2014) suggest that 66% intervals may be preferable to 95% confidence intervals because the latter produces intervals that may be too wide to be useful.

The approach we propose does not produce the uncertainty intervals by age and gender, as does the Bayes CCM approach described by Alkema *et al.* (2015), Yu *et al.* (2023, p. 934) and the CCR approach discussed by Swanson and Tayman (2014). The Bayes CCM approach also produces intervals for births, death, and migration. However, neither the Bayes CCM nor our approach take into account uncertainty in the input data themselves. However, as Yu *et al.* (2023, p. 934) implied, these are not likely to be among the most important sources of uncertainty for data in the United States and other countries where population forecasts are routinely produced.

In regard to our approach not providing uncertainty intervals by age and gender, Deming's (1950, pp. 127–134) "error propagation" was used to translate uncertainty in age group intervals found in the regression-based CCR forecasts reported by Swanson and Tayman (2014) to the total populations in question. In different forms, "error propagation" has been used by Alho and Spencer (2005), Espenshade and Tayman (1982), and Hansen, Hurwitz, and Madow (1953), among others. It may be possible to reverse-engineer error propagation and develop uncertainty measures by age and gender using our approach. The validity of this could be explored to determine if it is viable. As an approximation, one could generate age uncertainty intervals by controlling "low" and "high" numbers in a given forecast series to their corresponding

95% lower and upper limits, respectively, found using our proposed approach. The United Nations, for example, publishes not only probabilistic population forecasts but also a medium, low, and high series (United Nations, 2022).

Another possibility is to generate ARIMA forecasts of the ratio of land area to the population in a given age group for all age groups, which would generate probability intervals by age. These could be summed using the error propagation method to obtain a probability interval for the total population. In turn, this “bottom-up” result could be compared to the probability generated for the population as a whole.

6. Conclusion

Smith, Tayman, and Swanson (2001, p. 373) opined that future research would focus increasingly on measuring uncertainty in population forecasts and noted that while such research may not directly improve forecast accuracy, it will enhance our understanding of the uncertainty inherent in population forecasts. They went on to note that this change will imply a shift from “population projections” to “population forecasts,” a guideline we have followed in this paper. If one is trying to decide between a Bayesian approach to developing a probabilistic forecast and the approach described in this paper, the strengths and weaknesses discussed here need to be considered carefully and in the context of a particular forecasting environment: if it is resource-challenged, constrained by deadlines, and exposed to stakeholders who are not trained in statistical methods, the Bayesian approach may not be suitable.

In closing, we argue that the approach we propose and have described in this paper is well-suited to generate probabilistic world population forecasts and national and subnational population forecasts where CCM and CCR methods are routinely used to produce them. In the case of national and sub-national population projections, remember that migration will play a role, which, in conjunction with smaller populations, will likely lead to higher levels of uncertainty.

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Appendix

EXHIBIT 1. NCSS ARIMA (1,1,0) Report

Dataset	...\WORLD DENSITY 1950-2060.NCSS
Filter	YEAR<2021
Variable	DENSITY-TREND

Minimization Phase Section

Normal convergence.

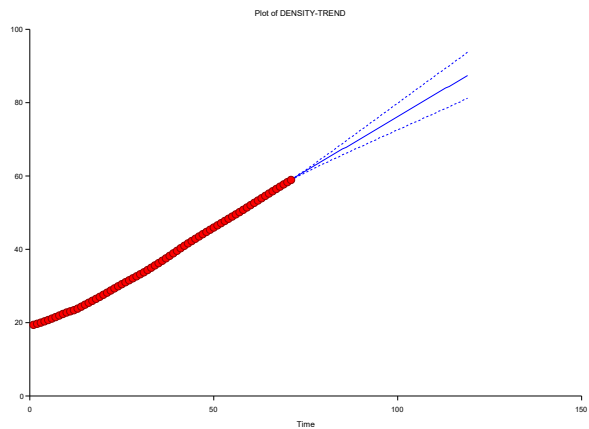
Model Description Section

Series	DENSITY-TREND
Model parameters)	Regular(1,1,0) Seasonal(No seasonal
Trend Equation	(16.53608)+(0.5865935)x(date)
Observations	71
Missing Values	None
Iterations	19
Pseudo R-Squared	99.999524
Residual Sum of Squares	0.04902241
Mean Square Error	0.0007104698
Root Mean Square	0.02665464

Model Estimation Section

Parameter Name	Parameter Estimate	Standard Error	T-Value	Prob Level
AR(1)	0.9672837	0.01841017	52.5407	0.000000

Forecast and Data Plot



Autocorrelation Plot Section

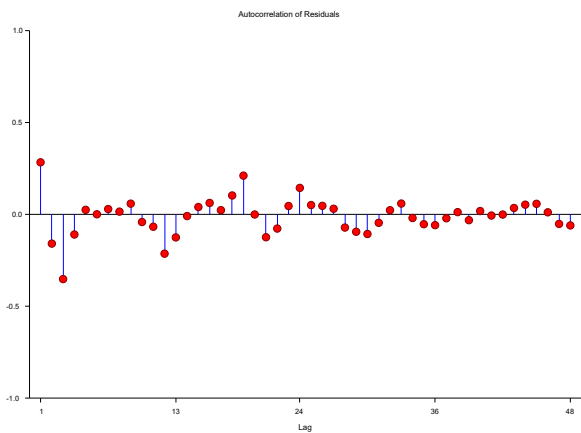


Table 1. Measures of Uncertainty for World Population Forecast, 2030–2060

Panel A. ARIMA Population Density Forecast											
2030			2040			2050			2060		
LL95%	Forecast	UL95%	LL95%	Forecast	UL95%	LL95%	Forecast	UL95%	LL95%	Forecast	UL95%
63.87	64.92	65.96	68.50	70.88	73.25	73.02	76.81	80.60	77.53	82.72	87.92

Panel B. ARIMA Upper and Lower 95% Bounds Relative to Forecast ^a											
2030			2040			2050			2060		
LL95%		UL95%	LL95%		UL95%	LL95%		UL95%	LL95%		UL95%
-0.01603		0.01603	0.03355		0.03355	-0.04938		0.04938	-0.06277		0.06277

Panel C. ARIMA 95% Intervals Applied to IDB Forecast (in Billions)											
2030			2040			2050			2060		
LL95%	IDB Forecast	UL95%	LL95%	IDB Forecast	UL95%	LL95%	IDB Forecast	UL95%	LL95%	IDB Forecast	UL95%
8.36	8.50	8.64	8.86	9.17	9.48	9.27	9.75	10.23	9.59	10.23	10.87

^a (LL95% - IDB Forecast) / IDB Forecast and (UL95% - IDB Forecast) / IDB Forecast.